Parabolic nonsingular integral operator on generalized Orlicz-Morrey spaces

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We study the boundedness of the parabolic nonsingular integral operator

$$\mathcal{R}f(x) = \int_{D_+^{n+1}} \frac{|f(y)|}{\rho(\widetilde{x} - y)^{n+2}}\, dy.$$

on parabolic generalized Orlicz-Morrey spaces $M^{\Phi,\phi}(D_+^{n+1})$. Here ρ is a certain metric on $\mathbb{R}_+^{n+1}$ and $\widetilde{x} = (x'', -x_n, t) \in D_+^{n+1} = \mathbb{R}^{n-1} \times \mathbb{R}_+ \times \mathbb{R}_+$. The generalized Orlicz-Morrey space $M^{\Phi,\phi}(D_+^{n+1})$ is equipped with the norm

$$\|f\|_{M^{\Phi,\phi}(D_+^{n+1})} \equiv \sup_{x \in D_+^{n+1},\ r>0} \frac{1}{\phi(x,r)} \Phi^{-1}\left(\frac{1}{|E^+(x,r)|}\right) \|f\|_{L^\Phi(E^+(x,r))},$$

where $E^+(x,r) = \left\{ y \in \mathbb{R}^{n+1} : \frac{|x'-y'|^2}{r^2} + \frac{|t-\tau|^2}{r^4} < 1 \right\}$.

The operator $\mathcal{R}$ and its commutator appear in connection with boundary estimates for solutions to parabolic equations.

One of the main theorems states that if Φ satisfies the so-called Δ_2 and ∇_2 condition and, together with $\phi_1, \phi_2 : D_+^{n+1} \times \mathbb{R}_+ \to \mathbb{R}_+$, satisfies some integral inequality, then the operator $\mathcal{R}$ is bounded from $M^{\Phi,\phi_1}(D_+^{n+1})$ to $M^{\Phi,\phi}(D_+^{n+1})$.

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