

# **Asymptotic behavior for the nonlinear damped wave equation with variable coefficients**

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In this work, we study the well-posedness of solutions, finite-time blow-up mechanism and asymptotic behavior of a nonlinear damped wave equation which arises from the propagation of elastic waves in the condensed matters. On the one hand, the well-posedness is intimately related to the parameter  $m$ , we construct exact counterexamples to show the finite-time blow-up phenomena, non-existence of the global solution with small initial data and prove the global existence of solutions with suitable  $m$ , etc. On the other hand, by the application of micro-local analysis, we explore the asymptotic energy estimates of the nonlinear equation which is influenced by two types of time-dependent oscillating coefficients on the principal Laplacian operator part. Furthermore, as an inverse problem, in order to demonstrate the optimality of the energy estimates, typical coefficients and initial Cauchy data will be constructed to show the lower bound of growth rate by the application of instability arguments.