On the effect of slowly decreasing initial data for nonlinear wave equations with damping and potential in the scaling critical regime

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In this talk, I'd like to present a recent result on the Cauchy problem for the nonlinear wave equation with damping and potential:

$$(\partial_t^2 + D(r)\partial_t - \Delta + V(r))U = \pm |U|^{p-1}U \quad \text{in } (0,T) \times \mathbb{R}^3,$$

where $p > 1.$ In my previous work obtained jointly with Prof. Georgiev and Prof. Wakasa, the coefficients of damping and potential terms are supposed to satisfy the relations: $V(r) = D(r)^2/4 - D(r)/2$ for $r > 0$, and $D(r) = 2/r$ for $r \geq 1.$ Without such restrictions, upper bounds of the lifespan of the solution to the above problem are derived by recent works due to Dai, Kubo, and Sobajima, and also Lai, Liu, Tu, and Wang. However, those proofs are based on the test function method and require the compactness of the support of the initial data. Here, instead of removing the relation between the coefficients of damping and potential terms, we relax the assumption on the initial data at spatial infinity. Actually, we obtain upper bound of the lifespan for slowly decreasing initial data, which seem to be optimal in comparison with the free case. Moreover, we are able to broaden the choice of the damping coefficient as $D(r) = \mu/r$ for $\mu \geq 0$ and $r \geq 1.$ The number μ affects on the shift of the critical exponent of the Strauss type.