On the solvability of the problem of synthesizing distributed and boundary controls in the optimization of oscillation processes.

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We study the solvability of the problem of synthesis of distributed and boundary controls in the optimization of oscillation processes described by partial integro-differential equations with the Fredholm integral operator. Functions of external and boundary actions are nonlinear with respect to the controls. For the Bellman functional, an integro-differential equation of a specific form is obtained and the structure of its solution is found, which allows this equation to be represented as a system of two equations of a simpler form. An algorithm for constructing a solution to the problem of synthesizing distributed and boundary controls described, and a procedure for finding the controls as a function (functional) of the state of the process is described.

Consider a controlled oscillatory process described by the boundary value problem

$$v_{tt} - Av = \lambda \int_0^T K(t,\tau)v(\tau,x)d\tau + f[t,x,u(t)], \quad x \in Q, \ \ 0 < t < T, \quad (1)$$

$$v(0,x) = \psi_1(x), \ \ v_t(0,x) = \psi_2(x), \ \ x \in Q, \quad (2)$$

$$\Gamma v(t,x) \equiv \sum_{i,k=1}^{n} a_{ik}(x) v_{x_k}(t,x) cos(\nu, x_i) + a(x)v(t,x) = p[t,x,\vartheta(t)],$$
$$x \in \gamma, 0 < t < T, \quad (3)$$

where A — is an elliptic operator

$$Av(t,x) = \sum_{i,k=1}^{n} (a_{ik} v_{x_k}(t,x))_{x_i} - c(x)v(t,x), \quad (4)$$

Q —is area of space R^n bounded by piecewise smooth curve γ ; $f[t,x,u(t)] \in H(Q_T)$, $\forall$ control $u(t) \in H(0,T)$, $p[t,x,\vartheta(t)] \in H(\gamma_T)$, $\forall$ of the boundary control $\vartheta(t) \in H(0,T)$; $H(Y)$ — is a Hilbert space of square-summable functions ; λ — is parameter; T — is fixed point in time; with respect to the function of external and boundary actions, we will assume that

$$f_u[t,x,u(t)] \neq 0, \ \ \forall (t,x) \in Q_T; \ \ p_\vartheta[t,x,\vartheta(t)] \neq 0, \ \ \forall (t,x) \in \gamma_T, \quad (5)$$

i.e., monotone with respect to the functional variable.

In the synthesis problem, it is required to find such controls $u^0(t) \in H(0,T)$ ¸ $\vartheta^0(t) \in H(0,T)$ which minimizes the integral quadratic functional.

$$J[u(t),\vartheta(t)] = \int_Q \{[v(T,x) - \xi_1(x)]^2 + [v_t(T,x) - \xi_2(x)]^2\} dx$$
$$+ \int_0^T \{\alpha M^2[t,u(t)] + \beta N^2[t,\vartheta(t)]\} dt, \ \alpha, \beta > 0, \quad (6)$$

defined on the set of generalized solutions of the boundary value problem (1)–(5).

In this case, the desired controls $u^0(t)$ and $\vartheta^0(t)$ defined as a function (functional) of the state of the controlled process, i.e. as

$$u^0(t) = u[t, v(t,x), v_t(t,x)], \ \ (t,x) \in Q_T, \ Q_T = Q \times (0,T),$$
$$\vartheta^0(t) = \vartheta[t, v(t,x), v_t(t,x)], \ \ (t,x) \in \gamma_T, \ \gamma_T = \gamma \times (0,T).$$