Generalized Sobolev-Morrey estimates for hypoelliptic operators on homogeneous groups

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Let $\mathbb{G} = \left(\mathbb{R}^N, \circ, \delta_\lambda\right)$ be a homogeneous group, Q is the homogeneous dimension of $\mathbb{G}$, $X_0, X_1, \ldots, X_m$ be left invariant real vector fields on $\mathbb{G}$ and satisfy Hörmander's rank condition on $\mathbb{R}^N$. Assume that $X_1, \ldots, X_m$ $(m \leq N-1)$ are homogeneous of degree one and X_0 is homogeneous of degree two with respect to the family of dilations $\left(\delta_\lambda\right)_{\lambda>0}$. Consider the following hypoelliptic operator $\mathcal{L} = \sum\limits_{i,j=1}^{m} a_{ij} X_i X_j + a_0 X_0$ with drift on $\mathbb{G}$, where (a_{ij}) is a $m \times m$ constant matrix satisfying the elliptic condition in $\mathbb{R}^m$ and $a_0 \neq 0$. In [1], for this class of operators, we obtain the generalized Sobolev-Morrey estimates by establishing boundedness of a large class of sublinear operators T_α, $\alpha \in [0, Q)$ generated by Calderón-Zygmund operators ($\alpha = 0$) and generated by fractional integral operator ($\alpha > 0$) on generalized Morrey spaces and proving interpolation results on generalized Sobolev-Morrey spaces on $\mathbb{G}$. The sublinear operators under consideration contain integral operators of harmonic analysis such as Hardy-Littlewood and fractional maximal operators, Calderón-Zygmund operators, fractional integral operators on homogeneous groups, etc.

Keywords: Hypoelliptic operators with drift, Homogeneous group, Fractional integral operator, Singular integral operators, Generalized Morrey space, Generalized Sobolev-Morrey estimates.

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