A generalised Samarskii–Ionkin type problem for the Laplace operator in a ball

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Let $x = (x_1, x_2, \dots, x_n) \in \mathbb{R}^n$ be an arbitrary point of the unit ball $\Omega = \{x = (x_1, x_2, \dots, x_n) \in \mathbb{R}^n : |x| < 1\} \subset \mathbb{R}^n$. Let $\alpha_k \in \{-1, 1\}$. Then $(\alpha_k)^2 = 1$. Denote $x^* = (-x_1, \alpha_2 x_2, \dots, \alpha_n x_n)$, and $\partial\Omega_+ \, (\partial\Omega_-)$ is a part of the sphere $\partial\Omega$, for which $x_1 > 0 \, (x_1 < 0)$. We also denote a part of the sphere $\partial\Omega$, for which $x_1 = 0$, by $\partial\Omega_0$.

In this talk we consider the following nonlocal boundary value problem for the Laplace operator in the ball, which is a multidimensional generalisation of the Samarskii-Ionkin problem.

The problem $S_{\alpha 1}$**.** *Find a function* $u(x) \in C^2(\Omega) \cap C^1(\bar{\Omega} \backslash \partial\Omega_0)$ *satisfying the Poisson's equation*

$$-\Delta u(x) = f(x), \quad x \in \Omega, \tag{1}$$

and the following boundary conditions

$$u(x) - \alpha u(x^*) = \tau(x), \quad x \in \partial\Omega_+, \tag{2}$$

$$\frac{\partial u(x)}{\partial n} - (-1)^k \frac{\partial u(x^*)}{\partial n} = \mu(x), \quad x \in \partial\Omega_+, \tag{3}$$

where $f(x) \in C^\varepsilon(\bar{\Omega})$*,* $\tau(x) \in C^{1+\varepsilon}[\partial\Omega_+], \mu(x) \in C^\varepsilon[\partial\Omega_+]$*,* $0 < \varepsilon < 1$*, and* α *is a fixed real number. Here,* $\frac{\partial}{\partial n}$ *is a derivative with respect to the direction of the outer normal to* $\partial\Omega$*.*

In the case when $\alpha = -(-1)^k$, we obtain periodic and antiperiodic boundary problems, which were studied earlier by M.Sadybekov and B.Turmetov.

In this talk we discuss the well-posedness and Fredholm property of the problem $S_{\alpha 1}$.