Spectral analysis of differential operator on compact Riemannian manifold

Karlygash Dosmagulova

Al-Farabi Kazakh National University
Kazakhstan
karlygash.dosmagulova@gmail.com

In this article we consider the case of the differential operator defined on the set of real or complex functions of a compact Riemannian manifold. The Laplacian is naturally generalized to the case of complex-valued functions in such a way that its spectrum in the complex and real cases coincides. Studying works where a detailed study of the structure of spaces of eigenfunctions in both cases is carried out, we were able to show specific examples. The invariance of the Laplace operator with respect to isometries implies the coincidence of the spectra of the Laplace operators of isometric Riemannian manifolds.

Thus, the spectrum of the Laplacian is an isometric invariant. One of the earliest results concerning the solution of the inverse problem belongs to G. Weil, who used the theory of integral equations developed by D. Hilbert to show that the volume of a bounded domain in Euclidean space can be determined from the asymptotic behavior of the eigenvalues of the Dirichlet boundary value problem for the Laplace operator. This particular result shows that the spectrum of the Laplacian contains information about some isometric invariants of the manifold on which it is given.

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