Spectrally invariant algebras of pseudodifferential operators with ultradifferentiable orbits

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Many operator algebras occurring in the theory of pseudodifferential operators are spectrally invariant. A famous example is the algebra of smooth operators T such that the orbit

$$G \ni x \mapsto \pi(x) T \pi(x)^{-1} \in \mathcal{L}(H_\pi) \tag{O}$$

is smooth for the Heisenberg group $G = \mathbb{H}$ and the Schrödinger representation $\pi = \rho$ on $H_\pi = L^2(\mathbb{R}^n)$. This algebra corresponds to the symbols in $\mathcal{S}^0_{0,0}(\mathbb{R}^n \times \mathbb{R}^n)$ via the Kohn-Nirenberg quantization. On compact Lie groups G an analogue situation can be found with respect the left regular representation $\pi = L$ on $H_\pi = L^2(G)$ and the symbol space $S^0_{0,0}(G \times \widehat{G})$. If G is a torus and $\pi = L$, it is known that the algebra of operators with analytic orbit (O) corresponds to a space of analytic symbols.

We discuss operator algebras with generalized regularity conditions on the orbits (O) and search for analogues of the cases above. In the case of $G = \mathbb{H}$ with $\pi = \rho$ or general compact Lie groups G with $\pi = L$ we get spectrally invariant algebras of pseudodifferential operators with ultradifferentiable orbits, which correspond to spaces of ultradifferentiable symbols.