On a non-local problem for the loaded parabolic-hyperbollic type equation with non-linear terms

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We consider a loaded parabolic-hyperbolic equation fractional order with nonlinear terms:

$$0=\begin{cases} u_{xx}-{}_CD_{ot}^{\alpha}u+a_1(x,\,t)\,u^{p_1}(x,\,t)+f_1(x,\,t;\,u(x,0)), \text{ for } t>0,\\ u_{xx}-u_{tt}+a_2(x,\,t)\,u^{p_2}(x,\,t)+f_2(x,\,t;\,u(x,\,0)), \text{ for } t<0,\end{cases} \tag{1}$$

where ${}_CD_{0t}^{\alpha}$ is Caputo differential operator, $a_i(x,\,t),\;\; f_i(x,\,t;\,u\,(x,\,0))$ are given functions, $p_i,\;\alpha$ are constants and $p_i>0,\;0<\alpha<1,\;i=1,\,2.$
Let $I\;=\;\{(x,t);t=0,\,0<x<l\}\,,\;\Omega_1\;=\;\{(x,t):0<x<l,\,0<t<h\}\,,$ $\Omega_2=\{(x,t):0<x+t<l,\,0<x-t<l,\;\;t<0\}$ and $\Omega=\Omega_1\cup I\cup\Omega_2$

Problem BV**.** It is required to find a solution $u\,(x,\,t)$ of the equation (1) with the following properties: 1) $u(x,t)\in C\,(\overline{\Omega})\cap C^2(\Omega_2),\,u_{xx},{}_CD_{ot}^{\alpha}u\in C\,(\Omega_1)\,,\,u_x\in C^1\big(\overline{\Omega}_1\setminus t=h\big)$; 2) $u(x,t)$ satisfy boundary value conditions:
$\alpha_1u(l,t)+\alpha_2u_x(l,t)=\varphi_1(t),\,\beta_1u(0,t)+\beta_2u_x(0,t)=\varphi_2(t),\,0\le t<h;$
$\gamma_1u\left(\frac{x}{2},-\frac{x}{2}\right)+\gamma_2u\left(\frac{x+l}{2},\frac{x-l}{2}\right)=\psi_1(x),\;0\le x\le l;$
and integral gluing condition:
$\lim\limits_{t\to+0}t^{1-\alpha}u_t(x,\,t)\;=\;\lambda_1(x)\,u_t(x,\,-0)\,+\,\lambda_2(x)\,u_x(x,\,-0)\;+\lambda_3(x)u\,(x,\,0)\,+\,\lambda_4(x),\;0\,<\,x\,<\,1$, where $\psi(x),\,\varphi_i(t)\$and\lambda_k(x)\,(k\,=\,\overline{1,\,4})$ are given continuous functions, such that $\sum\limits_{k=1}^{3}\lambda_k^2(x)\neq0,\,\alpha_i,\,\beta_i,\,\gamma_i,\;(i=1,2)$ are given constants.

Unique solvability of solution of the formulated problem was proved by the method of successive approximations of factorial law for Volterra type nonlinear integral equations, under certain conditions to the given functions.