Critical well-posedness for the modified Korteweg-de Vries equation and self-similar dynamics

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We consider the modified Korteweg-de Vries equation over the real line

$$u_t + u_{xxx} = \pm(u^3)_x.$$

This equation arises, for example, in the theory of water waves and vortex filaments in fluid dynamics. A particular class of solutions to (mKdV) are those which do not change under scaling transformations, the so-called *self-similar* solutions. Self-similar solutions blow-up when $t \to 0$ and determine the asymptotic behaviour of the evolution problem at $t = +\infty$.

The known local well-posedness results for the (mKdV) fail when one considers critical spaces, where the norm is scaling-invariant. This also means that self-similar solutions lie outside of the scope of these results. Consequently, the dynamics of (mKdV) around self-similar solutions are currently unknown.

In this talk, we will show existence and uniqueness of solutions to the (mKdV) lying on a critical space which includes both regular and self-similar solutions. Afterwards, we present several results regarding global existence, asymptotic behaviour at $t = +\infty$ and blow-up phenomena at $t = 0$.