Product formulas and convolutions for solutions of Sturm-Liouville equations

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The Fourier transform, which lies at the heart of the classical theory of harmonic analysis, is generated by the eigenfunction expansion of the Sturm-Liouville operator $-\frac{d^2}{dx^2}$. This naturally raises a question: *is it possible to generalize the main facts of harmonic analysis to integral transforms of Sturm-Liouville type?* In this talk we introduce a novel unified framework for the construction of product formulas and convolutions associated with a general class of regular and singular Sturm-Liouville boundary value problems. This unified approach is based on the application of the Sturm-Liouville spectral theory to the study of the associated hyperbolic equation. As a by-product, an existence and uniqueness theorem for degenerate hyperbolic Cauchy problems with initial data at a parabolic line is established. We will show that each Sturm-Liouville convolution gives rise to a Banach algebra structure in the space of finite Borel measures in which various probabilistic concepts and properties can be developed in analogy with the classical theory. We will discuss whether the convolution structure satisfies the basic axioms of the theory of hypergroups. Examples will be given, showing that many known convolution-type operators — such as the Hankel, Jacobi and Whittaker convolutions — can be constructed using this general approach.