Blow-up and lifespan estimate to a nonlinear wave equation in Schwarzschild spacetime

Ningan Lai
Department of Mathematics, Lishui University, 323000 Lishui, Zhejiang, PEOPLE'S REPUBLIC OF CHINA
hyayue@gmail.com

Luk (2013, Journal Eur. Math. Soc.) proved global existence for semilinear wave equations in Kerr spacetime with small angular momentum($a \ll M$)

$$\Box_{g_K}\phi = F(\partial\phi),$$

when the quadratic nonlinear term satisfies the null condition. In this work, we will show that if the null condition does not hold, at most we can have almost global existence for semilinear wave equations with quadratic nonlinear term in Schwarzschild spacetime, which is the special case of Kerr with $a = 0$

$$\Box_{g_S}\phi = (\partial_t\phi)^2,$$

where $\Box_{g_S}$ denotes the D'Alembert operator associated with Schwarzschild metric. What is more, if the power of the nonlinear term is replaced with p satisfying $3/2 \leq p < 2$, we still can show blow-up, no matter how small the initial data are. We do not have to assume that the support of the data should be far away from the event horizon.