Harmonic and Anharmonic Oscillators on the Heisenberg Group

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We propose a canonical version of the harmonic oscillator on the Heisenberg group $\mathbf{H}_n$ in terms of the representation theory of the Dynin-Folland group $\mathbf{H}_{n,2}$, a 3-step stratified Lie group, whose generic representations act on $L^2(\mathbf{H}_n)$. The classical relation between the harmonic oscillator on $\mathbb{R}^n$ and the sum of squares in the first stratum of the Heisenberg Lie algebra $\mathfrak{h}_n$ has an analog, which we turn into a definition: we define the harmonic oscillator on $\mathbf{H}_n$ as the image of the sub-Laplacian $\mathcal{L}_{\mathbf{H}_{n,2}}$ under the generic unitary irreducible representation π of the Dynin-Folland group which has formal dimension $d_\pi = 1$. More generally, this approach gives rise to a large class of so-called anharmonic oscillators by employing positive Rockland operators on $\mathbf{H}_{n,2}$. Employing methods developed by Elst and Robinson, we obtain spectral estimates for the harmonic and anharmonic oscillators on $\mathbf{H}_n$. The second part of the talk focuses on useful $L^{\mathtt{p}}$-$L^{\mathtt{q}}$-estimates for spectral multipliers of the sub-Laplacian $\mathcal{L}_{\mathbf{H}_{n,2}}$ and, more generally, of general Rockland operators on general graded groups. As a by-product, we recover Sobolev embeddings on graded groups, and obtain explicit hypoelliptic heat semigroup estimates. This is joint work with Prof. Michael Ruzhansky (Ghent University, Queen Mary University of London).