Uniqueness of standing-waves solutions to the non-linear Schroedinger equations for combined power-type non-linearities

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We illustrate recent results of orbital stability for non-linear dispersive equations. One of the problems we will be focusing on is the stability of standing-wave solutions to the non-linear Schrödinger equation

$$i\partial_t\varphi(t,x) + \Delta_x\varphi(t,x) - G(\varphi(t,x)) = 0.$$

The stability can be deduced from properties of the set of minima of the functional

$$E(u) = \frac{1}{2}\int_{\mathbb{R}^n} |\nabla u(x)|^2 dx + \int_{\mathbb{R}^n} G(u(x))dx$$

on the constraint $S = \{u \in H^1_{r,+}(\mathbb{R}^n) \mid \|u\|^2_{L^2} = 1\}$. The functions $H^1_{r,+}(\mathbb{R}^n)$ are radially symmetric (with respect to the origin), H^1 and positive. Orbital stability holds when there are finitely many minima. It is known from (M. K. Kwong, ARMA 1989) and (H. Berestycki and P. L. Lions, ARMA, 1983), that if $G(s) = -a|s|^p$, there is exactly one minimum. In this talk we present an extension of this uniqueness result to *combined power-type* non-linearities, $G(s) = -a|s|^p + b|s|^q$ and spatial dimension $n = 1$. We also provide a condition on G which guarantees that minima of the functional E on S are non-degenerate. As a consequence, there are only finitely many of them.