Harmonische Analysis — Blatt 4

*Wir Mathematiker und Physiker dürfen das stolze Bewußtsein hegen,
daß wir ein Wissensgebiet unser eigen nennen, welches der Menschheit
fortschreitend immer neuen äußeren Erfolg und innere Einsicht bietet,
und diese Freude an unserem Besitz, die müssen wir und wollen wir,
wenn sie uns je verloren gegangen sein sollte, wiedergewinnen!
(Felix Klein; 1849–1925)*

Problems

4.1. The matrix group
$$\mathrm{GL}(n,\mathbb{R}) = \{A \in \mathbb{R}^{n\times n} \;:\; \det A \neq 0\}$$
can be viewed as open subset of $\mathbb{R}^{n^2}$ and is locally compact with the induced topology. Determine left and right Haar measure on this group and deduce that this group is unimodular.
(Hint: Suppose the Haar measure is absolutely continuous with respect to the n^2-dimensional Lebesgue measure. What does the left invariance of the measure imply for its density?)

Topics as preparation

4.2. The Gelfand theory was only developed for unital commutative C^*-algebras. We already showed that any C^*-algebra without unit element can be embedded into a C^*-algebra with unit element in an essentially unique way. Use this to develop a Gelfand theory for C^*-algebras without unit. What are the major differences?

Source: Section 1.3 of G.B. Folland: *A Course on Abstract Harmonic Analysis*

4.3. Summability properties of Fourier and Laplace series.

To any continuous and periodic function $f \in \mathrm{C}(\mathbb{R}/\mathbb{Z})$ we can associate a formal Fourier series
$$\sum_{k\in\mathbb{Z}} \alpha_k \mathrm{e}^{2\pi\mathrm{i}kt} \qquad \text{with} \qquad \alpha_k = \int_0^1 f(t)\mathrm{e}^{-2\pi\mathrm{i}kt}\mathrm{d}t.$$
In general, this series does not converge in the Banach space $\mathrm{C}(\mathbb{R}/\mathbb{Z})$. The aim of this topic is to discuss summability properties of these series and similar series in terms of spherical functions on spheres $\mathbb{S}^n$ in the framework of homogeneous Banach spaces.

Source: Results for Laplace series can be found in the paper

H. Berens, P. Butzer, S. Pawelke: *Limitierungsverfahren von Reihen mehrdimensionaler Kugelfunktionen und deren Saturationsverhalten*, Publications RIMS 1968 Vol. 4 pages 201–268

results for Fourier series are classical and can be taken from Y. Katznelson *Introduction to Harmonic Analysis*, Chapter I.2