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Fachbereich Mathematik
Fakultät Mathematik und Physik
Universität Stuttgart
Pfaffenwaldring 57
D-70 569 Stuttgart

E-Mail: preprints@mathematik.uni-stuttgart.de
WWW: http://www.mathematik.uni-stuttgart.de/preprints

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THE INFLUENCE OF SURFACE TENSION AND CONFIGURATIONAL FORCES ON THE STABILITY OF LIQUID-VAPOR INTERFACES

B. KABIL & C. ROHDE *

Abstract

The stability of liquid-vapor interfaces in a multidimensional van der Waals fluid is analyzed. We consider interfaces which connect liquid and vapor states as subsonic shock waves. Surface tension and configurational forces in the form of a kinetic relation determine the evolution of the interface.
Stability results for the interface in the sense of energy estimates for solutions of the linearized problem are given. The normal mode analysis of the problem shows that in particular the uniform Kreiss-Lopatinskiĭ condition is satisfied as long as surface tension and amount of energy dissipation are positive but remain small.
The analysis relies on [6], where surface tension is arbitrary but energy dissipation is zero. Non-stability results for the same system without surface tension and without energy dissipation can be found in [3].

Keywords: Liquid-vapor interface, Kreiss-Lopatinskiĭ condition, uniform stability, Kreiss symmetrizer, energy estimate.

1 Introduction

We consider a system of evolution equations which describes the motion of an isothermal compressible fluid which appears in a liquid and a vapor phase. The motion of the fluid in the bulk phase is modeled by the Euler equations (see (2.1.1), (2.1.2) below). At the interface the mass conservation law and a dynamical version of the Young-Laplace law for momentum balance hold (see (2.2.1), (2.2.2)). We consider as interfaces subsonic phase transitions. In this case it is well-known that one more condition has to be added to ensure well-posedness [1, 20].
Together with curvature and surface tension it drives as a configurational force the dynamics of the interface. We choose as additional condition a kinetic relation as in [9] ((2.3.2) below). In this way solutions of the resulting free boundary problem satisfy the second law of thermodynamics while the amount of energy dissipation is exactly prescribed. The kinetic relation depends solely on the relative mass flux across the interface. For more details on the modeling background see [1, 9, 10].

We are interested in the stability of propagating liquid-vapor interfaces in the sense of proving energy estimates for the solutions of the linearized system.
The standard approach to analyze stability of compressive (Laxian) shocks can be found in e.g. [14]. In our case, this method cannot be applied directly for undercompressive shock waves since curved sharp liquid-vapor interfaces cannot be seen as weak solutions of the Euler equations.

*Institute of Applied Analysis and Numerical Simulation, University of Stuttgart, Pfaffenwaldring 57, 70569 Stuttgart, Germany.
E-mail: Bugra.Kabil@mathematik.uni-stuttgart.de, Christian.Rohde@mathematik.uni-stuttgart.de

The dynamical stability analysis of (curved) liquid-vapor interfaces using energy estimates has been initiated by Benzoni-Gavage [2, 3] and by Benzoni-Gavage & Freistühler [6]. In passing we note, that in [2] the stable subspace of the linearized Euler equations is given explicitly, a result which we use in our work. In [2], neither surface tension nor kinetic relations are considered. So-called surface waves can occur and uniform stability is not achieved. We mention that the existence of surface waves is not necessarily contradicting uniform stability, see e.g. [6] for a result in this direction. For a detailed study on the different types of surface waves we refer to [12]. The work [3] treats the interface problem with small amount of energy dissipation but neglecting surface tension. Here the so-called Lopatinskiĭ determinant has no roots on the right complex half plane. Especially central neutral modes do not exist. In [6], the system without dissipation but with surface tension is studied.

The major issue of this paper is to study the stability of solutions of the linearized system with small but positive surface tension *and* small but positive energy disspation. The main result of this work is presented in Theorem 8 where the stability result is given by energy estimates for the linear constant coefficients version of the free boundary problem.

The derivation of the energy estimate is based on the so-called Lopatinskiĭ determinant of the evolution operator for the ODE system associated via Fourier-Laplace transformation with the linearized free boundary problem, see Section 3. In this matter, we follow the usual methods, e.g. [5, 6, 13, 7, 15]. The behavior of the roots of the Lopatinskiĭ determinant is crucial. If this function has no zeros on the right complex half plane, it will be possible to get energy estimates. We show by a careful perturbation analysis that this function does not vanish for small surface tension and small dissipation on the right complex half plane. We note that in our case the so-called uniform Kreiss-Lopatinskiĭ (UKL) condition is satisfied, such that it is possible to construct a Kreiss symmetrizer to get energy estimates for the solutions of the system with respect to the initial data. The method to construct Kreiss symmetrizers is based on so-called normal mode analysis.

The paper is organized as follows. In Section 2 we reformulate the system as an Initial Boundary Value Problem and introduce the equations including dissipation and surface tension. In Section 3 we define the Lopatinskiĭ determinant and so we derive the Kreiss-Lopatinskiĭ condition for the problem. Then we explain how to construct a Kreiss symmetrizer and how to get the energy estimates for the problem.

2 The Mathematical Model

2.1 Bulk Equations

We consider the motion of an ideal compressible fluid in $\mathbb{R}^d$ with constant temperature, for $d > 1$. The system of equations is given for space variable $x = (x_1, ..., x_d) \in \mathbb{R}^d$, time variable $t > 0$, unknown density $\rho = \rho(x,t) > 0$ and velocity $\mathbf{u}(x,t) = (u_1(x,t), ..., u_d(x,t)) \in \mathbb{R}^d$ by the Euler equations

$$\begin{aligned} \rho_t + \nabla \cdot (\rho \mathbf{u}) &= 0, & (2.1.1)\\ (\rho \mathbf{u})_t + \nabla \cdot (\rho \mathbf{u} \otimes \mathbf{u}) + \nabla p(\rho) &= 0. & (2.1.2) \end{aligned}$$

In this system p denotes the pressure of the fluid. We choose a nonmonotone pressure law $p = p(\rho)$ such that there are constants $l^* > v^* > 0$ (see figure 1):

$$\begin{cases} p'(\rho) > 0, & \text{if} \quad 0 < \rho < v^* \qquad \text{(vapor states)},\\ p'(\rho) < 0, & \text{if} \quad v^* < \rho < l^* \qquad \text{(spinodial states)},\\ p'(\rho) > 0, & \text{if} \quad l^* < \rho \qquad \text{(liquid states)}. \end{cases}$$

For example, one can choose for the pressure the van der Waals law which is given in the form

$$p(\rho) = \frac{RT\rho}{1 - b\rho} - a\rho^2, \qquad (2.1.3)$$

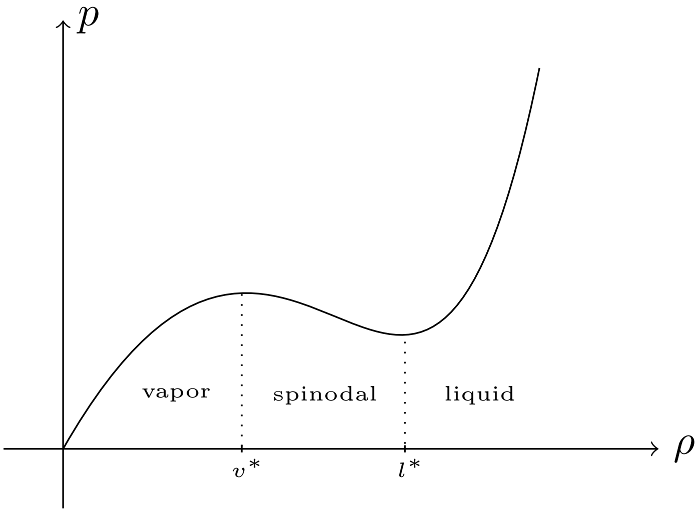

Figure 1: Pressure law

where $R > 0$ is the perfect gas constant and $T > 0$ is the temperature. The constants $a \geq 0$ and $b > 0$ are given and depend on the fluid. For T below a critical temperature the van der Waals pressure behaves as in figure 1.

We consider fluid states outside of the spinodal region. Under this assumption the system (2.1.1), (2.1.2) becomes hyperbolic. Such assumption on pressure yields outside the spinodal region the coexistence of liquid and vapor phases, where we observe propagating phase boundaries, e.g. [2, 3]. These propagating phase boundaries are discontinuous solutions of the Euler equations.
In this work we study the stability of the linearized equations obtained by (2.1.1), (2.1.2). We consider piecewise smooth solutions of (2.1.1), (2.1.2) in the following sense (see figure 2). There exist a smooth hypersurface $\Sigma(t)$ and two smooth functions $(\rho^+, \mathbf{u}^+)$ and $(\rho^-, \mathbf{u}^-)$ with either ρ^+ (ρ^-) in the liquid (vapor) region or ρ^- (ρ^+) in the vapor (liquid) region defined on respective domains $V_+(t)$ and $V_-(t)$ on either side of the hypersurface $\Sigma(t)$ such that

$$\begin{aligned} \rho_t^\pm + \nabla\cdot(\rho^\pm \mathbf{u}^\pm) &= 0, \qquad \text{in} \qquad V_\pm(t), && (2.1.4)\\ (\rho^\pm \mathbf{u}^\pm)_t + \nabla\cdot(\rho^\pm \mathbf{u}^\pm \otimes \mathbf{u}^\pm) + \nabla p^\pm &= 0, \qquad \text{in} \qquad V_\pm(t). && (2.1.5) \end{aligned}$$

In the sequel we fix the jump conditions across $\Sigma(t)$.

2.2 Mechanical Jump Conditions

In the standard theory the Rankine-Hugoniot jump conditions associated with equations (2.1.1), (2.1.2) are given by

$$\begin{aligned} [\rho(\mathbf{u}\cdot\mathbf{n} - \sigma)] &= 0, && (2.2.1)\\ [\rho(\mathbf{u}\cdot\mathbf{n} - \sigma)\mathbf{u} + p\mathbf{n}] &= (d-1)\kappa s \mathbf{n}, && (2.2.2) \end{aligned}$$

where the brackets denote the jump across the interface

$$[f] = \lim_{\varepsilon \searrow 0} f(x + \varepsilon\mathbf{n}) - f(x - \varepsilon\mathbf{n})$$

for any $x \in \Sigma(t)$, $s > 0$ the surface tension, κ the mean curvature, $\mathbf{n} \in \mathbb{R}^d$ the unit normal vector to the moving interface in x and $\sigma \in \mathbb{R}$ the normal speed of propagation of the interface in x. For the

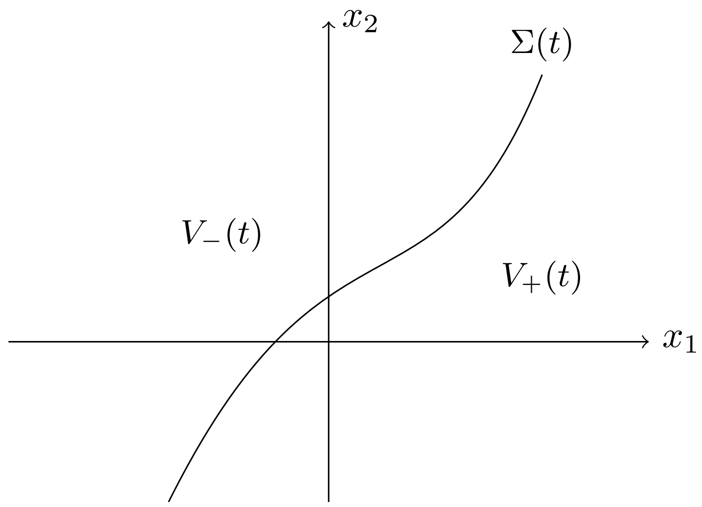

Figure 2: A smooth hypersurface $\Sigma(t)$

details on the model we refer to [8].

We define the mass transfer flux as

$$\begin{aligned} j &:= \lim_{\varepsilon \searrow 0} \left(\rho(x - \varepsilon \mathbf{n})(\mathbf{u}(x - \varepsilon \mathbf{n}) \cdot \mathbf{n} - \sigma)\right) \\ &= \lim_{\varepsilon \searrow 0} \left(\rho(x + \varepsilon \mathbf{n})(\mathbf{u}(x + \varepsilon \mathbf{n}) \cdot \mathbf{n} - \sigma)\right). \end{aligned}$$

As in [2], a phase transition in liquid-vapor interfaces is rather similar to an undercompressive shock, see also [11]. That is why, we have to define one more jump condition which is physically seen comprehensible. We introduce the chemical potential g, where we have $\mathrm{d}_\rho \, p(\rho) = c^2$ and $\mathrm{d}_\rho \, e(\rho) = g$. In this case c denotes the sound speed and $e = \rho E$, where E is the specific free energy. As it was used in the literature like [6, 2, 3, 4, 19], we can take the kinetic relation without dissipation

$$\left[g + \frac{j^2}{2\rho^2} \right] = 0. \tag{2.2.3}$$

The system with the kinetic relation (2.2.3) without dissipation was studied in [6], where surface waves exist. Surface waves which are explained in [2] can destroy the stability estimates. But it was shown by construction of modified Kreiss symmetrizers that one can get also energy estimates. It is mentioned that the uniform Kreiss-Lopatinskiĭ condition is not satisfied in this case. We consider the system with dissipation, that means we have to change equation (2.2.3). This will be done by using a kinetic relation proposed in [9], [10].

2.3 Kinetic Relation

The kinetic relation which we consider is assumed to be a relation between the mass transfer flux and the driving force, see [9]. We denote the jump $\left[g + \frac{j^2}{2\rho^2} \right]$ as the driving force. The aim is to put a kinetic relation to describe subsonic phase transitions, see [3]. The following dissipation inequality shows the change of entropy dissipation (see [9])

$$j \cdot \left[g + \frac{j^2}{2\rho^2} \right] \leq 0. \tag{2.3.1}$$

The simplest ansatz to satisfy the dissipation inequality is to use the following relation

$$\left[g + \frac{j^2}{2\rho^2} \right] = -\mathrm{B} j, \tag{2.3.2}$$

where $\mathrm{B} > 0$ is the so-called interfacial mobility constant. We choose equation (2.3.2) as the added kinetic relation of the system. When $\mathrm{B} = 0$, we have the case as in [6] where dissipation was neglected. Especially, entropy dissipation of the system is presented for the case $\mathrm{B} \neq 0$, see [9, 10]. The ansatz shows that the left side of the dissipation inequality is strictly negative for nonzero mass transfer flux j. Altogether we have the jump conditions

$$\begin{aligned} [\rho(\mathbf{u}\cdot\mathbf{n} - \sigma)] &= 0, && (2.3.3)\\ [\rho(\mathbf{u}\cdot\mathbf{n} - \sigma)\mathbf{u} + p\mathbf{n}] &= (d-1)\kappa s \mathbf{n}, && (2.3.4)\\ \left[g + \frac{j^2}{2\rho^2} \right] &= -\mathrm{B} j. && (2.3.5) \end{aligned}$$

Remark 1. *Another possible kinetic relation as done in [3] is the following*

$$\left[g + \frac{j^2}{2\rho^2} \right] = -\nu\, j \int_{-\infty}^{+\infty} v'(\xi)^2 \, \mathrm{d}\,\xi, \tag{2.3.6}$$

where ν is a positive constant and v is the solution to the viscous capillary profile equation $v'' = \nu j v' + p(\frac{1}{\rho_r}) + j^2 \frac{1}{\rho_r} - p(v) - j^2 v$ with $v(\pm\infty) = \frac{1}{\rho_{r,l}}$, where $\rho_{r,l}$ are given constants, see Section 3. We note that one can get by linearization of (2.3.6) *with $B = \alpha(j)\nu + o(\nu)$ the kinetic relation* (2.3.2)*, see [3].*

2.4 Reformulation of the Jump Conditions

We assume that the interface $\Sigma(t)$ is almost flat in the sense that $\Sigma(t)$ can be written for $X \in C^2\left(\mathbb{R}^{d-1} \times [0,\infty)\right)$ in the following form

$$\Sigma(t) = \left\{ \mathbf{x} = (x_1, \dots, x_d) \;\middle|\; x_d = X(x_1, \dots, x_{d-1}, t) \right\}.$$

In the sequel the gradient operator with respect to all components except the last component is given by

$$\check{\nabla} = (\partial_{x_1}, \dots, \partial_{x_{d-1}})^{\mathsf{T}},$$

and we define $y = (x_1, \dots, x_{d-1})$. Then, we can write the unit normal vector $\mathbf{n}$ in terms of X as follows

$$\mathbf{n} := \frac{1}{\sqrt{1 + \|\check{\nabla} X\|^2}} \left(-\check{\nabla} X, 1 \right)^{\mathsf{T}}, \tag{2.4.1}$$

which yields

$$\sigma = \frac{\partial_t X}{\sqrt{1 + \|\check{\nabla} X\|^2}}. \tag{2.4.2}$$

The curvature κ can also be given in terms of X as

$$\kappa = \frac{1}{d-1} \check{\nabla} \cdot \left(\frac{\check{\nabla} X}{\sqrt{1 + \|\check{\nabla} X\|^2}} \right). \tag{2.4.3}$$

We consider now the hyperplane $\left\{\mathbf{x} \in \mathbb{R}^d \,\middle|\, x_d = 0\right\}$ and decompose the velocity $\mathbf{u} = (\mathbf{v}, u)$, where $\mathbf{v}$ denotes the tangential part and u its normal part with respect to this hyperplane.
With these notations we write all jump conditions (2.3.3)-(2.3.5) in terms of X as

$$\begin{aligned}
\left[\rho\,(u - \partial_t X - \mathbf{v}\cdot\check{\nabla}X)\right] &= 0, && (2.4.4)\\
\left[\rho\,(u - \partial_t X - \mathbf{v}\cdot\check{\nabla}X)\mathbf{v} - p\check{\nabla}X\right] &= 0, && (2.4.5)\\
\left[\rho\,(u - \partial_t X - \mathbf{v}\cdot\check{\nabla}X)u + p\right] &= s\check{\nabla}\cdot\check{\nabla}X, && (2.4.6)\\
\left[\left(1 + \|\check{\nabla}X\|^2\right)g + \tfrac{1}{2}(u - \partial_t X - \mathbf{v}\cdot\check{\nabla}X)^2\right] &= -\mathrm{B}j\left(1 + \|\check{\nabla}X\|^2\right). && (2.4.7)
\end{aligned}$$

3 Kreiss-Lopatinksiĭ Condition

The aim of this section is to derive the so-called Lopatinskiĭ determinant. The Lopatinskiĭ determinant, $\Delta : (\tau, \eta) \mapsto \Delta(\tau, \eta)$, depends on frequency $\tau \in \mathbb{C}$ and wave number $\eta \in \mathbb{R}^{d-1}$. If this function does not vanish on the right complex half plane, this will be equivalent to the UKL condition. If there are only roots for $\mathrm{Re}\tau = 0$ on the right complex half plane, the UKL condition won't be satisfied. As done in [2, 3] we consider a planar dynamic interface as a reference interface. That means, we consider a weak solution of (2.1.4)-(2.1.5) and (2.3.3)-(2.3.5) with $\kappa = 0$ of the form

$$(\rho^{\pm}, \mathbf{u}^{\pm}) = \begin{pmatrix} \rho_{r,l} \\ \mathbf{v}_{r,l} \\ u_{r,l} \end{pmatrix}, \tag{3.1}$$

where $(\rho_{r,l}, \mathbf{v}_{r,l}, u_{r,l})$ is a constant vector. We consider subsonic phase transitions. Define the Mach number

$$M_{r,l} := \frac{|\mathbf{u}_{r,l}\cdot\mathbf{n} - \sigma|}{c_{r,l}} > 0$$

for $c_{r,l}^2 = p'(\rho_{r,l})$. In our case, we require for subsonic phase transitions that

$$0 < M_{r,l} < 1 \tag{3.2}$$

is satisfied. Further, we note that we assume

$$(u_r - u_l)(c_r - c_l) < 0 \tag{3.3}$$

to ensure entropy dissipation. We can assume without loss of generality that the mass transfer flux

$$j = \rho_l(\mathbf{u}_l\cdot\mathbf{n} - \sigma) = \rho_r(\mathbf{u}_r\cdot\mathbf{n} - \sigma)$$

is positive. Further we assume that the right and left tangential parts of the solutions and also the normal speed of the interface are zero, that is

$$\mathbf{v}_r = \mathbf{v}_l = 0, \qquad \sigma = 0.$$

This can also be done without loss of generality due to Galilean invariance. Altogether, we study the stability of planar interfaces with respect to nonplanar perturbations.

3.1 The Linearized System

The next step is to linearize the system (2.1.4), (2.1.5). As it is done in [2, 3, 6], we are going to linearize the system (2.1.4), (2.1.5) about the special solution (3.1). We plug

$$(\rho^{\pm}(x,t), \mathbf{v}^{\pm}(x,t), u^{\pm}(x,t)) = (\rho_{r,l}\,,\, \mathbf{v}_{r,l},\, u_{r,l}\,) + \delta\cdot(\rho_{\pm}(x,t), \mathbf{v}_{\pm}(x,t), u_{\pm}(x,t))$$

for $\delta > 0$ and some perturbation functions $(\rho_\pm(x,t), \mathbf{v}_\pm(x,t), u_\pm(x,t))$ into the equations (2.1.4), (2.1.5), differentiate with respect to δ and evaluate in $\delta = 0$. Then we obtain with $c_{r,l}^2 = p'(\rho_{r,l}) > 0$ the linearized system

$$
\begin{aligned}
\partial_t \rho_\pm + \rho_{r,l} \check{\nabla} \cdot \mathbf{v}_\pm \pm \rho_{r,l} \partial_z u_\pm \pm u_{r,l} \partial_z \rho_\pm &= 0, \quad \text{in} \quad z > 0, && (3.1.1)\\
\partial_t \mathbf{v}_\pm \pm u_{r,l} \partial_z \mathbf{v}_\pm + \frac{c_{r,l}^2}{\rho_{r,l}} \check{\nabla} \rho_\pm &= 0, \quad \text{in} \quad z > 0, && (3.1.2)\\
\partial_t u_\pm \pm u_{r,l} \partial_z u_\pm \pm \frac{c_{r,l}^2}{\rho_{r,l}} \partial_z \rho_\pm &= 0, \quad \text{in} \quad z > 0. && (3.1.3)
\end{aligned}
$$

This is a system of $2(d+1)$ first order partial differential equations in the half space. Note that we have transformed the coordinates to

$$z = \pm(x_d - X(y,t)).$$

Let $\mathbf{U} := (\rho_-, \mathbf{v}_-, u_-, \rho_+, \mathbf{v}_+, u_+)$, and $\mathbf{L} := \mathbf{L}(\partial_t, \nabla)$ be the vector valued differential operator, such that the linearized system (3.1.1), (3.1.3) can be written for $z > 0$ in the form

$$\mathbf{L}[\mathbf{U}] = 0.$$

Further we obtain $d+2$ boundary conditions at $z = 0$ by linearizing the boundary conditions (2.4.4)-(2.4.7) about $(\rho_{r,l}, \mathbf{v}_{r,l}, u_{r,l})$ and $X = 0$. The linearized conditions are the following

$$
\begin{aligned}
u_r\, \rho_+ + \rho_r\, u_+ - u_l\, \rho_- - \rho_l\, u_- - [\rho]\, \partial_t X &= 0, && (3.1.4)\\
\rho_r\, u_r\, \mathbf{v}_+ - \rho_l\, u_l\, \mathbf{v}_- - [p]\, \check{\nabla} X &= 0, && (3.1.5)\\
(c_r^2 + u_r^2)\, \rho_+ + 2\rho_r\, u_r\, u_+ - (c_l^2 + u_l^2)\, \rho_- - 2\rho_l\, u_l\, u_- &= s \check{\nabla} \cdot \check{\nabla} X, && (3.1.6)\\
\frac{c_r^2}{\rho_r}\, \rho_+ + u_r\, u_+ - \frac{c_l^2}{\rho_l}\, \rho_- - u_l\, u_- - [u]\, \partial_t X & \\
+ \mathrm{B}\, \rho_l\, u_- + \mathrm{B}\, u_l\, \rho_- - \mathrm{B}\, \rho_l\, \partial_t X &= 0. && (3.1.7)
\end{aligned}
$$

We can also write the boundary conditions for $z = 0$ in the following way

$$\mathbf{b}[X] + \mathbf{M}\mathbf{U} = 0, \tag{3.1.8}$$

where $\mathbf{M} \in \mathbb{R}^{2(d+1) \times 2(d+1)}$ is a matrix and $\mathbf{b}$ is given by

$$\mathbf{b}(\partial_t, \check{\nabla}) = \begin{pmatrix} -[\rho]\partial_t \\ -[p]\check{\nabla} \\ -s\, \check{\nabla} \cdot \check{\nabla} \\ -\left([u] + \mathrm{B}\rho_l\right) \partial_t \end{pmatrix}. \tag{3.1.9}$$

Remark 2. *We have mentioned in Remark 1 that one can also consider the kinetic relation* (2.3.6)*. By linearizing one can get the same equation as the one in [3] in the following way*

$$
\begin{aligned}
\frac{c_r^2}{\rho_r}\, \rho_+ + u_r\, u_+ - \frac{c_l^2}{\rho_l}\, \rho_- - u_l\, u_- - [u]\, \partial_t X & \\
+ \tilde{\nu}\, (\rho_l\, u_- + u_l\, \rho_- - \rho_l\, \partial_t X) &= 0.
\end{aligned}
$$

where $\tilde{\nu} = \alpha(j)\nu + o(\nu)$, see for details [3].

3.2 Determination of the Lopatinskiĭ Determinant

In the general theory, one derives the Kreiss-Lopatinskiĭ condition by looking for particular solutions of the problem (3.1.1)-(3.1.7) of the form

$$\mathbf{w}(y,z,t) = e^{\tau t + \mathrm{i}\eta\cdot y}\, \mathbf{W}(z),$$

where $\eta \in \mathbb{R}^{d-1}$, $\tau \in \mathbb{C}$ and $\mathbf{W} : \mathbb{R}^+ \to \mathbb{R}^{d+1}$ is some function. This procedure called Normal Mode Analysis means to search solutions of (2.1.4), (2.1.5) and (2.3.3)-(2.3.5) that could contradict well-posedness. For that, we are interested in vectors

$$(R_-, \mathbf{V}_-, U_-, R_+, \mathbf{V}_+, U_+),$$

which belong to the stable invariant subspace $\mathcal{E}^s(\tau, \eta)$ [1] of the ordinary differential equations obtained by inserting the particular solution in equations (3.1.1)-(3.1.3), where the components of the particular solution are given for $\chi \in \mathbb{C}$ in the following form

$$\begin{aligned}
\rho_\pm(y, z, t) &= e^{\tau t + \mathrm{i}\eta\cdot y}\, R_\pm(z), && (3.2.1)\\
\mathbf{v}_\pm(y, z, t) &= e^{\tau t + \mathrm{i}\eta\cdot y}\, \mathbf{V}_\pm(z), && (3.2.2)\\
u_\pm(y, z, t) &= e^{\tau t + \mathrm{i}\eta\cdot y}\, U_\pm(z), && (3.2.3)\\
X(y, t) &= e^{\tau t + \mathrm{i}\eta\cdot y}\, \chi. && (3.2.4)
\end{aligned}$$

Substituting this ansatz in (3.1.4)-(3.1.7) yields the algebraic system

$$\begin{aligned}
u_r\, R_+ + \rho_r\, U_+ - u_l\, R_- - \rho_l\, U_- - [\rho]\, \tau\, \chi &= 0, && (3.2.5)\\
\rho_r\, u_r\, \mathbf{V}_+ - \rho_l\, u_l\, \mathbf{V}_- - \mathrm{i}[p]\, \chi\, \eta &= 0, && (3.2.6)\\
(c_r^2 + u_r^2)\, R_+ + 2\rho_r\, u_r\, U_+ - (c_l^2 + u_l^2)\, R_- - 2\rho_l\, u_l\, U_- &= -s\|\eta\|^2\, \chi, && (3.2.7)\\
\frac{c_r^2}{\rho_r}\, R_+ + u_r\, U_+ - \frac{c_l^2}{\rho_l}\, R_- - u_l\, U_- - [u]\, \tau\, \chi & && \\
+ \mathrm{B}\, \rho_l\, U_- + \mathrm{B}\, u_l\, R_- - \mathrm{B}\, \rho_l\, \tau\, \chi &= 0. && (3.2.8)
\end{aligned}$$

The stable subspace $\mathcal{E}^s(\tau, \eta)$ was studied already in [3, Lemma 3], where one can find the proof of the following lemma.

Lemma 3. *Assume that there exist $\eta \in \mathbb{R}^{d-1}$, $\tau \in \mathbb{C}$ with $\mathrm{Re}\,\tau > 0$, such that (3.2.1)-(3.2.3) are solutions of equations (3.1.1)-(3.1.3) with $R_\pm(+\infty) = 0$, $\boldsymbol{V}_\pm(+\infty) = 0$ and $U_\pm(+\infty) = 0$.*

Then the vector

$$(R_-, \boldsymbol{V}_-, U_-)$$

has to be parallel for $z \geq 0$ to the vector

$$(\rho_l\, (\tau - u_l\, \omega_l), -\mathrm{i}c_l^2\eta, c_l^2\omega_l), \tag{3.2.9}$$

while

$$(R_+, \boldsymbol{V}_+, U_+)$$

has to be orthogonal to

$$(\omega_r(c_r^2 - u_r^2) + \tau u_r, \mathrm{i}\rho_r u_r \eta, -\rho_r \tau). \tag{3.2.10}$$

In equations (3.2.9), (3.2.10) the complex wave numbers $\omega_{r,l}$ are defined by the solutions of the following dispersion relation

$$c_{r,l}^2(\omega_{r,l}^2 - \|\eta\|^2) = (\tau - u_{r,l}\, \omega_{r,l})^2,$$

with $\mathrm{Re}\,\omega_l < 0$ and $\mathrm{Re}\,\omega_r > 0$.

We abbreviate by introducing

$$a_{r,l} := \omega_{r,l}(c_{r,l}^2 - u_{r,l}^2) + \tau u_{r,l}.$$

Now we derive an algebraic system which is equivalent to (3.2.5)-(3.2.8) and define the Lopatinskiĭ determinant. We use the form of the stable invariant subspace such that we can simplify equations (3.2.5)-(3.2.8) to a system written in matrix form as done in [6].

[1] A stable invariant subspace of a matrix A with n rows, n columns and entries in $\mathbb{C}$ is formed of vectors $v \in \mathbb{C}^n$ such that $(\exp(tA))v$ tends to zero as $t \to \infty$ and the decay is exponentially fast.

Lemma 4. *The vector* $(R_-, \boldsymbol{V}_-, U_-, R_+, \boldsymbol{V}_+, U_+, \chi)$ *is a nonzero solution of the equations* (3.2.5)-(3.2.8) *and belongs to the stable subspace* $\mathcal{E}^s(\tau,\eta)$ *if and only if there exists a complex number* $\zeta \in \mathbb{C}$ *such that* $(\zeta\rho_l, R_+, \boldsymbol{V}_+, U_+, \chi)$ *is nonzero and*

$$\begin{pmatrix} -\tau[\rho] & u_r & \rho_r & a_l \\ \|\eta\|^2[p] & -a_r & \rho_r\tau & u_l c_l^2\|\eta\|^2 \\ s\|\eta\|^2 & u_r^2+c_r^2 & 2\rho_r u_r & u_l a_l + c_l^2\tau \\ -\tau([u]+\mathrm{B}\rho_l) & \frac{c_r^2}{\rho_r} & u_r & \tau\frac{c_l^2}{\rho_l} - \mathrm{B}a_l \end{pmatrix} \begin{pmatrix} \chi \\ R_+ \\ U_+ \\ -\zeta\rho_l \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}.$$

Definition 5. *The Lopatinskiĭ determinant* $\Delta : \mathbb{C}\times\mathbb{R}^+ \to \mathbb{C}$ *is defined as*

$$\Delta(\tau,\|\eta\|; s, \mathrm{B}) := \begin{vmatrix} -\tau[\rho] & u_r & \rho_r & a_l \\ \|\eta\|^2[p] & -a_r & \rho_r\tau & u_l c_l^2\|\eta\|^2 \\ s\|\eta\|^2 & u_r^2+c_r^2 & 2\rho_r u_r & u_l a_l + c_l^2\tau \\ -\tau([u]+\mathrm{B}\rho_l) & \frac{c_r^2}{\rho_r} & u_r & \tau\frac{c_l^2}{\rho_l} - \mathrm{B}a_l \end{vmatrix}. \tag{3.2.11}$$

Remark 6. *For the case* $\eta = 0$, *the Lopatinskiĭ determinant does not vanish, i.e.*

$$\Delta(\tau, 0) \neq 0,$$

as in [2, 3, 6].

We write the Lopatinskiĭ determinant as a polynomial of the form

$$\Delta(\tau,\|\eta\|; s,\mathrm{B}) = p_1(\tau,\|\eta\|) + s\,p_2(\tau,\|\eta\|) + \mathrm{B}\,p_3(\tau,\|\eta\|) + s\mathrm{B}\,p_4(\tau,\|\eta\|), \tag{3.2.12}$$

where p_i are given polynomials. The properties of the roots of p_1, p_1+sp_2 and $p_1+\mathrm{B}p_3$ are studied in [2, 3, 6] and summarized below. We want to use these results to make statements about the roots of $\Delta(\tau,\|\eta\|)$ for small s and small B.

In the literature the following quantities

$$V := \frac{\tau}{\mathrm{i}\|\eta\|}, \qquad W_l := \frac{a_l}{\mathrm{i}\|\eta\|c_l}, \qquad W_r := \frac{a_r}{\mathrm{i}\|\eta\|c_r}$$

are used to simplify the Lopatinskiĭ determinant. The results for the polynomials p_1+sp_2 and $p_1+\mathrm{B}p_3$ were derived with these quantities, which do not change the arguments for the roots with respect to τ and η.

The Lopatinskiĭ determinant is then given as follows

$$\begin{aligned} \Delta(\tau,\|\eta\|; s,\mathrm{B}) \quad &:= \quad \mathrm{i}\|\eta\|^3\frac{c_l}{u_r^2 u_l}\left(j\varphi_1(\tau,\|\eta\|) + \mathrm{i}s\|\eta\|u_r u_l\varphi_2(\tau,\|\eta\|) + \mathrm{B}\varphi_3(\tau,\|\eta\|)\right) \\ &\quad\;\; + s\mathrm{B}\varphi_4(\tau,\|\eta\|). \end{aligned} \tag{3.2.13}$$

The functions $\varphi_1, \varphi_2, \varphi_3$ and φ_4 are given by

$$\begin{aligned} \varphi_1(\tau,\|\eta\|) &= -[u]^2\left(c_r V - u_r W_r\right)\left(c_l c_r V^2 + u_r u_l W_l W_r\right), \\ \varphi_2(\tau,\|\eta\|) &= \begin{vmatrix} u_r & 1 & W_l \\ c_r W_r & V & -u_l c_l \\ c_r^2 u_r & u_r^2 & u_l c_l V \end{vmatrix}, \\ \varphi_3(\tau,\|\eta\|) &= jV\left(c_r V - u_r W_r\right)\left(c_l u_r W_r + c_r u_l W_l\right) \\ &\quad\; - j u_r[u]\left(c_r^2 - u_r^2\right)\left(c_l V - u_l W_l\right), \\ \varphi_4(\tau,\|\eta\|) &= -a_l\|\eta\|^2\left(\rho_r u_r \tau + a_r\rho_r\right). \end{aligned}$$

We summarize the properties of the polynomials p_1+sp_2 and $p_1+\mathrm{B}p_3$ in the following.

Case: $s = \mathrm{B} = 0$

In this case we have the polynomial $\Delta(\tau, \|\eta\|; 0, 0) = p_1(\tau, \|\eta\|)$ as the Lopatinskiĭ determinant of the system. It was shown in [2] that p_1 does not vanish for $\mathrm{Re}\,\tau > 0$ but there are zeros with $\mathrm{Re}\,\tau = 0$. One has also surface waves in this case, see for the details [2]. The Kreiss-Lopatinskiĭ condition is satisfied but not the uniform one. The polynomial p_1 has the form

$$p_1(\tau, \|\eta\|) = c_1\tau^3 + c_2\tau^2\boldsymbol{\eta} + c_3\tau\boldsymbol{\eta}^2 + c_4\boldsymbol{\eta}^3,$$

where the coefficients c_1, c_2, c_3 and c_4 are real constants and where we have used $\boldsymbol{\eta}$ instead of $\|\eta\|$.

Case: $s > 0, \quad \mathrm{B} = 0$

The Lopatinskiĭ determinant becomes $\Delta(\tau, \|\eta\|; s, 0) = p_1(\tau, \|\eta\|) + s\,p_2(\tau, \|\eta\|)$. It was shown in [6, Theorem 3.2] that $p_1 + s\,p_2$ does not vanish for $\mathrm{Re}\,\tau > 0$ but there are zeros with $\mathrm{Re}\,\tau = 0$ for a small range, for a given η_0. For $\boldsymbol{\eta} > \eta_0$, the polynomial $p_1 + s\,p_3$ does not vanish for $\mathrm{Re}\,\tau \geq 0$. In other words, for $\mathrm{Re}\,\tau \geq 0$ and $\boldsymbol{\eta} > \eta_0$ we have $p_1 + s\,p_2 \neq 0$. It is noted that the uniform Kreiss-Lopatinskiĭ condition is not satisfied. The polynomial has the form

$$\begin{aligned} p_1(\tau, \|\eta\|) + s\,p_2(\tau, \|\eta\|) \quad &= \quad c_1\tau^3 + c_2\tau^2\boldsymbol{\eta} + c_3\tau\boldsymbol{\eta}^2 + c_4\boldsymbol{\eta}^3 \\ &\qquad + s\left(c_2^s\tau^2\boldsymbol{\eta}^2 + c_3^s\tau\boldsymbol{\eta}^3 + c_4^s\boldsymbol{\eta}^4\right), \end{aligned}$$

where all the coefficients c_2^s, c_3^s and c_4^s are real constants.

Case: $s = 0, \quad \mathrm{B} > 0$

In this case we obtain the polynomial $\Delta(\tau, \|\eta\|; 0, \mathrm{B}) = p_1(\tau, \|\eta\|) + \mathrm{B}\,p_3(\tau, \|\eta\|)$. It was shown in [3] that $p_1 + \mathrm{B}\,p_3$ does not vanish for $\mathrm{Re}\,\tau \geq 0$ and for small $\mathrm{B} > 0$. There exists a constant $\mathrm{B}_0 > 0$ such that for all $\mathrm{B} \in (0, \mathrm{B}_0)$, $\mathrm{Re}\,\tau \geq 0$ and $\eta \in \mathbb{R}^{d-1}$, we have that

$$p_1 + \mathrm{B}\,p_3 \neq 0.$$

The polynomial has the form

$$\begin{aligned} p_1(\tau, \|\eta\|) + \mathrm{B}\,p_3(\tau, \|\eta\|) \quad &= \quad c_1\tau^3 + c_2\tau^2\boldsymbol{\eta} + c_3\tau\boldsymbol{\eta}^2 + c_4\boldsymbol{\eta}^3 \\ &\qquad + \mathrm{B}\left(c_1^{\mathrm{B}}\tau^3 + c_2^{\mathrm{B}}\tau^2\boldsymbol{\eta} + c_3^{\mathrm{B}}\tau\boldsymbol{\eta}^2 + c_4^{\mathrm{B}}\boldsymbol{\eta}^3\right), \end{aligned}$$

where the coefficients $c_1^{\mathrm{B}}, c_2^{\mathrm{B}}, c_3^{\mathrm{B}}$ and c_4^{B} are also real constants.

The aim is now to show that $\Delta(\tau, \|\eta\|; s, \mathrm{B}) = p_1 + s\,p_2 + \mathrm{B}\,p_3 + s\mathrm{B}\,p_4$ defined by (3.2.12) does not vanish in $\mathrm{Re}\,\tau \geq 0$ and $\eta \in \mathbb{R}^{d-1}$ for small s and small B. We will show this in two steps. In the first step we consider the polynomial for all $\boldsymbol{\eta} \leq \eta_0$, where η_0 is given as in [6, Theorem 3.2]. The second step is to consider the other values.
We also take the perturbation constant $\mathrm{B}_0 > 0$ from [3, Theorem 1]. It is possible that we have B smaller as $\mathrm{B}_0 > 0$ in [3]. Then we will take the minimum of these two constants. We get the following theorem for the roots of the Lopatinskiĭ determinant.

Theorem 7. *Let $\Delta(\tau, \|\eta\|)$ be defined by* (3.2.11). *Assume that the numbers $u_{r,l}$ and $c_{r,l}$ satisfy the inequalities* (3.2), (3.3).
Then there are constants $s_0 > 0$ and $\mathrm{B}_0 > 0$ depending continuously on $u_{r,l}$ and $c_{r,l}$, such that for all $s \in (0, s_0)$ and $\mathrm{B} \in (0, \mathrm{B}_0)$, we have for $\mathrm{Re}\,\tau \geq 0$ and $\eta \in \mathbb{R}^{d-1}$

$$\Delta(\tau, \|\eta\|) \neq 0.$$

Proof. We will use the information about the polynomials derived above. First we consider the determinant for fixed small $\boldsymbol{\eta} \leq \eta_0$. We consider the polynomial

$$\Delta(\tau, \|\eta\|; s, \mathrm{B}) = (p_1 + \mathrm{B}p_3) + s \cdot (p_2 + \mathrm{B}\, p_4)\,. \tag{3.2.14}$$

We know from [3, Theorem 1] that $\Delta(\tau, \|\eta\|; 0, \mathrm{B}) = p_1 + \mathrm{B}p_3$ with these assumptions does not vanish for $\mathrm{Re}\,\tau \geq 0$ and for some $\mathrm{B} \in (0, \mu_0)$ with $\mu_0 > 0$. Let λ^{B} be a root of $p_1 + \mathrm{B}p_3$ and fix $\mathrm{B} > 0$. Differentiating equation (3.2.14) with respect to s yields

$$\frac{\partial}{\partial \tau}(p_1 + \mathrm{B}p_3)\,(\lambda(s)) \cdot \lambda'(s) + (p_2 + \mathrm{B}p_4)(\lambda(s)) + s \cdot \lambda(s) \cdot \frac{\partial}{\partial \tau}(p_2 + \mathrm{B}p_4)\,(\lambda(s)) = 0.$$

Evaluating at $s = 0$ implies for the root $\lambda(s)$ of Δ and for $\frac{\partial}{\partial \tau}(p_1 + \mathrm{B}p_3)\ (\lambda^{\mathrm{B}}) \neq 0$

$$\lambda'(0) = -\frac{(p_2 + \mathrm{B}\, p_4)\ (\lambda^{\mathrm{B}})}{\frac{\partial}{\partial \tau}(p_1 + \mathrm{B}p_3)\ (\lambda^{\mathrm{B}})}.$$

The case for $\frac{\partial}{\partial \tau}(p_1 + \mathrm{B}p_3)\ (\lambda^{\mathrm{B}}) = 0$ is clear, since λ^{B} is then also a root of $p_2 + \mathrm{B}\, p_4$ and thus a root of Δ. The roots depend continuously on s, so we conclude that the root λ of Δ is in a neighborhood of $\lambda(0) := \lambda^{\mathrm{B}}$, i.e. $|\lambda - \lambda^{\mathrm{B}}| = \mathcal{O}(s)$, see figure 3 and for more details [18]. If we choose s small enough, we obtain that the real part of λ is still strictly negative, because the real part of $\lambda(0)$ was negative.

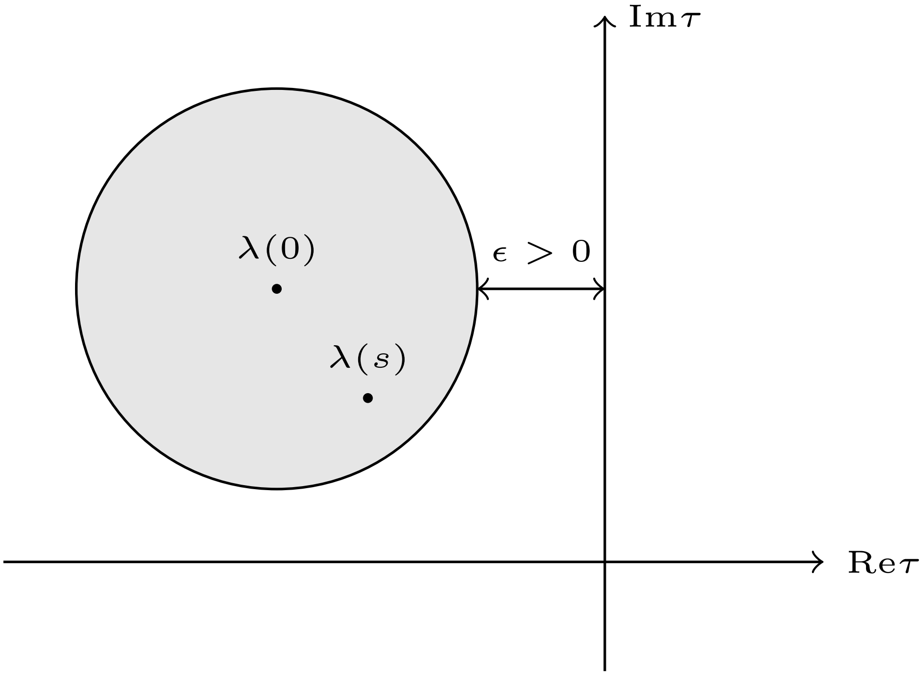

Figure 3: Ball around a perturbed root $\lambda(s)$ in the complex halfplane. The radius of the ball depends on the perturbation factor s.

Second, we consider the determinant for $\boldsymbol{\eta} > \eta_0$. We consider again the polynomial

$$\Delta(\tau, \|\eta\|; s, \mathrm{B}) = (p_1 + s\, p_2) + \mathrm{B} \cdot (p_3 + s p_4)\,.$$

We know from [6, Theorem 3.2] that $\Delta(\tau, \|\eta\|; s, 0) = p_1 + s\, p_2$ does not vanish for $\mathrm{Re}\,\tau \geq 0$ and for all $s > 0$. We fix s. The same arguments as above show that the perturbed root with respect to B is located in a ball around the root of the polynomial $p_1 + s\,p_2$. Altogether we obtain for small s and B that for $\operatorname{Re}\tau \geq 0$ and $\eta \in \mathbb{R}^{d-1}$

$$\Delta(\tau, \|\eta\|) \neq 0.$$

□

We remark that the polynomial Δ as the determinant of the matrix (3.2.11) above is well-defined for $\eta \in \mathbb{R}^{d-1}$ and $\operatorname{Re}\tau \geq 0$. It is also analytic in (τ, η), see Remark 6.

4 Stability

The aim of this section is to get weighted energy estimates for the solutions of the linearized system (3.1.1)-(3.1.7). A standard way to derive energy estimates for this kind of problems relies on so-called Kreiss symmetrizers.

4.1 Linearized System and Weighted Estimates

We consider the linearized system (3.1.1)-(3.1.3). We use again the abbreviations $\mathbf{U} := (\rho_-, \mathbf{v}_-, u_-, \rho_+, \mathbf{v}_+, u_+)$ and $\mathbf{L} = \mathbf{L}(\partial_t, \nabla)$. The linearized system has for $z > 0$ the form

$$\mathbf{L}[\mathbf{U}] = 0. \tag{4.1.1}$$

We write the boundary conditions (3.1.4)-(3.1.7) in the form (3.1.8). The operator $\mathbf{b}$ is given by

$$\mathbf{b}(\partial_t, \check{\nabla}) = \begin{pmatrix} -[\rho]\partial_t \\ -[p]\check{\nabla} \\ -s\,\check{\nabla}\cdot\check{\nabla} \\ -\left([u] + \mathrm{B}\rho_l\right)\partial_t \end{pmatrix}. \tag{4.1.2}$$

The associated nonhomogeneous problem is given by

$$\begin{aligned} \mathbf{L}[\mathbf{U}] &= \mathbf{f} \qquad \text{for} \quad z > 0, &\quad (4.1.3)\\ \mathbf{b}[X] + \mathbf{M}\mathbf{U} &= \mathbf{g} \qquad \text{for} \quad z = 0. &\quad (4.1.4) \end{aligned}$$

We also use weighted-in-time functions, namely,

$$\tilde{\mathbf{U}} = \exp(-\gamma t)\cdot\mathbf{U} \qquad \text{and} \qquad \tilde{\mathrm{X}} = \exp(-\gamma t)\cdot\mathrm{X}$$

as well as

$$\tilde{\mathbf{f}} = \exp(-\gamma t)\cdot\mathbf{f} \qquad \text{and} \qquad \tilde{\mathbf{g}} = \exp(-\gamma t)\cdot\mathbf{g},$$

for $\gamma > 0$. Using weighted functions we consider the following modified system. It is easy to see, that $\tilde{\mathbf{U}}$, $\tilde{\mathrm{X}}$ satisfy the equations

$$\begin{aligned} \mathbf{L}_\gamma[\tilde{\mathbf{U}}] &= \tilde{\mathbf{f}} \qquad \text{for} \quad z > 0, &\quad (4.1.5)\\ \mathbf{b}_\gamma[\tilde{\mathrm{X}}] + \mathbf{M}\tilde{\mathbf{U}} &= \tilde{\mathbf{g}} \qquad \text{for} \quad z = 0, &\quad (4.1.6) \end{aligned}$$

where $\mathbf{L}_\gamma = \mathbf{L} + \gamma$ and $\mathbf{b}_\gamma$ is given by replacing ∂_t by $\gamma + \partial_t$ in $\mathbf{b}$.

Now we specify the stability results in the sense of energy estimates for the system (4.1.5)-(4.1.6) in the following main theorem.

Theorem 8 (Energy Estimate)**.** *Assume that* (3.2)*,* (3.3) *hold and* s_0 *and* B_0 *are given by Theorem 7.*

Then for all $\mathrm{B} \in (0, \mathrm{B}_0)$, $s \in (0, s_0)$ *and* $\gamma_0 > 0$, *there exists a constant* $C > 0$ *such that for all* $\gamma \geq \gamma_0$ *and all solutions*

$$(\widetilde{\boldsymbol{U}}, \widetilde{X}) \in C^1(\mathbb{R}^+, L^2(\mathbb{R}^d)) \times H^1(\mathbb{R}^d) \tag{4.1.7}$$

of (4.1.5)-(4.1.6) with $\widetilde{\boldsymbol{f}}, \widetilde{\boldsymbol{g}} \in L^2$ *the following inequality holds*

$$\gamma \| \widetilde{\boldsymbol{U}} \|^2_{L^2(\mathbb{R}^d \times \mathbb{R}^+)} + \| \widetilde{\boldsymbol{U}}(0) \|^2_{L^2(\mathbb{R}^d)} + \| \widetilde{X} \|^2_{H^1_\gamma(\mathbb{R}^d)} \tag{4.1.8}$$

$$\leq C \left(\| \widetilde{\boldsymbol{g}} \|^2_{L^2(\mathbb{R}^d)} + \frac{1}{\gamma} \| \widetilde{\boldsymbol{f}} \|^2_{L^2(\mathbb{R}^d \times \mathbb{R}^+)} \right) . \tag{4.1.9}$$

We note that the space $H^s_\gamma(\mathbb{R}^d)$ stands for the usual Sobolev space equipped with weighted norms

$$\|u\|^2_{H^s_\gamma(\mathbb{R}^d)} = \int\limits_{\mathbb{R}^d} \left(\gamma^2 + \|\eta\|^2 + \delta^2\right)^s |\widehat{u}(\eta, \delta)|^2 \, \mathrm{d}\,\eta \, \mathrm{d}\,\delta \,,$$

where $\widehat{u}(\eta, \delta)$ is the Fourier transform of u in (y, t).

The proof of Theorem 8 is based on Kreiss symmetrizers. The standard way is to take the Fourier transform of the equations (4.1.5)-(4.1.6) and symmetrizing the equations by the Kreiss symmetrizer. We take the Fourier transform for the problem (4.1.5)-(4.1.6) in $(\mathbf{y}, t)$, where we denote the Fourier variable as (η, δ). We obtain the system

$$\mathbf{L}_\gamma(\mathrm{i}\delta, \mathrm{i}\eta)\widehat{\mathbf{U}} = \widehat{\mathbf{f}} \qquad \text{for} \quad z > 0, \tag{4.1.10}$$

$$\mathbf{b}_\gamma(\mathrm{i}\delta, \mathrm{i}\eta)\widehat{X} + \mathbf{M}\widehat{\mathbf{U}} = \widehat{\mathbf{g}} \qquad \text{for} \quad z = 0, \tag{4.1.11}$$

where $\widehat{\mathbf{U}}, \widehat{\mathrm{X}}, \widehat{\mathbf{f}}$ and $\widehat{\mathbf{g}}$ denote the Fourier transforms of $\tilde{\mathbf{U}}, \tilde{\mathrm{X}}, \tilde{\mathbf{f}}$ and $\tilde{\mathbf{g}}$. We can also write this system in the form

$$\mathbf{L}(\gamma + \mathrm{i}\delta, \mathrm{i}\eta)\widehat{\mathbf{U}} = \widehat{\mathbf{f}} \qquad \text{for} \quad z > 0, \tag{4.1.12}$$

$$\mathbf{b}(\gamma + \mathrm{i}\delta, \mathrm{i}\eta)\widehat{X} + \mathbf{M}\widehat{\mathbf{U}} = \widehat{\mathbf{g}} \qquad \text{for} \quad z = 0. \tag{4.1.13}$$

It is noted that we will write τ instead of $\gamma + \mathrm{i}\delta$ in what follows. The system (4.1.12)-(4.1.13) represents a system of ordinary differential equations. We can write the system (4.1.12) in the form

$$\frac{\mathrm{d}\,\widehat{\mathbf{U}}}{\mathrm{d}\,z} = \mathbf{A}(\tau, \mathrm{i}\eta)\widehat{\mathbf{U}} + \mathbf{A}_z^{-1}\widehat{\mathbf{f}}, \tag{4.1.14}$$

where the matrices $\mathbf{A}(\tau, \mathrm{i}\eta)$ and $\mathbf{A}_z^{-1}$ are given. Especially the matrix $\mathbf{A}(\tau, \mathrm{i}\eta)$ has a block diagonal structure which allows us to construct a Kreiss symmetrizer, see [16].

4.2 Kreiss Symmetrizer

A Kreiss symmetrizer is a uniformly bounded and matrix-valued C^∞-mapping $(\tau, \eta) \mapsto \mathbf{K}(\tau, \eta)$ on $D := \{(\tau, \eta) \in \mathbb{C} \times \mathbb{R}^{d-1} \big| \operatorname{Re}\tau \geq 0, \, |\tau| + |\eta| \neq 0\}$ such that there are constants $c_0, c_1, c_2 > 0$ with

$$\mathbf{K}\mathbf{A} + \mathbf{A}^*\mathbf{K}^* \geq c_0(\operatorname{Re}\tau)\,\mathrm{I} \qquad \text{in} \quad D, \tag{4.2.1}$$

$$\mathbf{K} + c_1\Sigma^*\Sigma \geq c_2\mathrm{I} \qquad \text{in} \quad D. \tag{4.2.2}$$

Here we used the notation in the sense of bilinear forms and $\Sigma := \mathbf{P}\mathbf{M}$ where $\mathbf{P}$ is the orthogonal projection on $\mathbf{b}^\perp$. The projection can be written as

$$\mathbf{P} := \mathrm{I} - \frac{\mathbf{b} \otimes \mathbf{b}}{\|\mathbf{b}\|^2},$$

where I is the identity.

In this part we will use a Kreiss symmetrizer for the problem as in [13], so that we can estimate the solutions of the linearized system. The general theory about Kreiss symmetrizers can be found in [5, 6, 13].

The fact that the uniform Kreiss-Lopatinskiĭ condition is satisfied (Theorem 7) allows us to construct this kind of symmetrizer, see for details [6, 5, 15, 7, 13]. Especially the construction is based on three steps, see [6]. The first step is to build so-called local symmetrizers in boundary points $\operatorname{Re}\tau = 0$, the second step is to collect these symmetrizers to a global one and the last step is to find suitable constants such that (4.2.2) holds. In our case we have a non homogeneous boundary condition. The boundary conditions are quasi-homogeneous which also allows us as in [6] the construction of a Kreiss symmetrizer. We note that $\mathbf{b}$ is quasi-homogeneous in the sense of introducing a variable $\zeta := \|\eta\|^2$ such that $\mathbf{b}(\tau, \mathrm{i}\eta, \zeta)$ is homogeneous.

Now we assume that $\widehat{\mathbf{U}} \in H^1(\mathbb{R}^+)$ is a solution of the following problem

$$\frac{\mathrm{d}\,\widehat{\mathbf{U}}}{\mathrm{d}\,z} = \mathbf{A}(\tau, \mathrm{i}\eta) \cdot \widehat{\mathbf{U}} + \mathbf{A}_z^{-1} \cdot \widehat{\mathbf{f}}, \qquad z > 0, \tag{4.2.3}$$

$$\Sigma \cdot \widehat{\mathbf{U}} = \mathbf{P} \cdot \widehat{\mathbf{g}}, \qquad z = 0. \tag{4.2.4}$$

The first step is to multiply the equation (4.2.3) with $\widehat{\mathbf{U}}^*\mathbf{K}$ from the left. This implies

$$\widehat{\mathbf{U}}^*\mathbf{K}\,\frac{\mathrm{d}\,\widehat{\mathbf{U}}}{\mathrm{d}\,z} = \widehat{\mathbf{U}}^*\mathbf{K}\,\mathbf{A}\,\widehat{\mathbf{U}} + \widehat{\mathbf{U}}^*\mathbf{K}\,\mathbf{A}_z^{-1}\,\widehat{\mathbf{f}}.$$

We take the real part of this equation and obtain

$$\frac{1}{2}\frac{\mathrm{d}}{\mathrm{d}\,z}\left(\widehat{\mathbf{U}}^*\mathbf{K}\,\widehat{\mathbf{U}}\right) = \operatorname{Re}\left(\widehat{\mathbf{U}}^*\mathbf{K}\,\mathbf{A}\,\widehat{\mathbf{U}}\right) + \operatorname{Re}\left(\widehat{\mathbf{U}}^*\mathbf{K}\,\mathbf{A}_z^{-1}\,\widehat{\mathbf{f}}\right).$$

Further we get

$$\operatorname{Re}\left(\widehat{\mathbf{U}}^*\mathbf{K}\,\mathbf{A}\,\widehat{\mathbf{U}}\right) = \frac{1}{2}\,\widehat{\mathbf{U}}^*\left(\mathbf{K}\mathbf{A} + \mathbf{A}^*\mathbf{K}^*\right)\widehat{\mathbf{U}},$$

and this equation implies

$$\frac{\mathrm{d}}{\mathrm{d}\,z}\left(\widehat{\mathbf{U}}^*\mathbf{K}\,\widehat{\mathbf{U}}\right) = \widehat{\mathbf{U}}^*\left(\mathbf{K}\mathbf{A} + \mathbf{A}^*\mathbf{K}^*\right)\widehat{\mathbf{U}} + 2\,\operatorname{Re}\left(\widehat{\mathbf{U}}^*\mathbf{K}\,\mathbf{A}_z^{-1}\,\widehat{\mathbf{f}}\right).$$

Integrating on $\mathbb{R}^+$ yields

$$-\left(\widehat{\mathbf{U}}^*(0)\mathbf{K}\,\widehat{\mathbf{U}}(0)\right) = \left\langle \widehat{\mathbf{U}}^*,\, \left(\mathbf{K}\mathbf{A} + \mathbf{A}^*\mathbf{K}^*\right)\widehat{\mathbf{U}}\right\rangle + 2\operatorname{Re}\left\langle \widehat{\mathbf{U}}^*,\, \mathbf{K}\,\mathbf{A}_z^{-1}\,\widehat{\mathbf{f}}\right\rangle.$$

Using the properties of the Kreiss symmetrizer (4.2.1), (4.2.2), we obtain the following inequality for a generic constant $c > 0$ and all $\varepsilon > 0$

$$c_0\,\operatorname{Re}\tau\,\|\widehat{\mathbf{U}}\|^2_{L^2(\mathbb{R}^+)} + c_2\|\widehat{\mathbf{U}}(0)\|^2 \le c_1\|\widehat{\mathbf{g}}\|^2 + c\left(\frac{1}{\varepsilon}\|\widehat{\mathbf{f}}\|^2_{L^2(\mathbb{R}^+)} + \varepsilon\|\widehat{\mathbf{U}}\|^2_{L^2(\mathbb{R}^+)}\right).$$

We choose $\varepsilon = \frac{c_0\,\operatorname{Re}\tau}{2c}$ so that we can absorb the last term on the right side. That means, we obtain

$$\frac{c_0\,\operatorname{Re}\tau}{2}\,\|\widehat{\mathbf{U}}\|^2_{L^2(\mathbb{R}^+)} + c_2\|\widehat{\mathbf{U}}(0)\|^2 \le c_1\|\widehat{\mathbf{g}}\|^2 + \frac{2c^2}{c_0\operatorname{Re}\tau}\,\|\widehat{\mathbf{f}}\|^2_{L^2(\mathbb{R}^+)}.$$

Altogether we obtain for a generic constant c

$$\operatorname{Re}\tau\,\|\widehat{\mathbf{U}}\|^2_{L^2(\mathbb{R}^+)} + \|\widehat{\mathbf{U}}(0)\|^2 \le c\left(\|\widehat{\mathbf{g}}\|^2 + \frac{1}{\operatorname{Re}\tau}\,\|\widehat{\mathbf{f}}\|^2_{L^2(\mathbb{R}^+)}\right). \tag{4.2.5}$$

Using Plancherel's theorem we get an estimate in weighted norms

$$
\begin{aligned}
\operatorname{Re}\tau\,\|e^{-\gamma t}\mathbf{U}\|^2_{L^2(\mathbb{R}^d\times\mathbb{R}^+)} + \|e^{-\gamma t}\mathbf{U}(0)\|^2_{L^2(\mathbb{R}^d)} & \qquad (4.2.6)\\
\leq c\left(\|e^{-\gamma t}\mathbf{g}\|^2_{L^2(\mathbb{R}^d)} + \frac{1}{\operatorname{Re}\tau}\,\|e^{-\gamma t}\mathbf{f}\|^2_{L^2(\mathbb{R}^d\times\mathbb{R}^+)}\right), & \qquad (4.2.7)
\end{aligned}
$$

where we note that $(y,t,z)\in\mathbb{R}^{d-1}\times\mathbb{R}\times\mathbb{R}^+$.

4.3 Estimate for the Interface

We estimate the unknown front X in a quite technical way. We have

$$
\mathbf{b}(\tau,\mathrm{i}\eta) = \begin{pmatrix} -[\rho]\tau \\ -[p]\mathrm{i}\eta \\ s\|\eta\|^2 \\ -\left([u]+\mathrm{B}\rho_l\right)\tau \end{pmatrix} \qquad (4.3.1)
$$

and

$$
\mathbf{b}(\tau,\mathrm{i}\eta)\widehat{X} + \mathbf{M}\widehat{\mathbf{U}} = 0, \qquad (4.3.2)
$$

for $z=0$. As done in [6] one can construct a Kreiss symmetrizer with the properties (4.2.1) and (4.2.2).

We know from (4.1.13)

$$
\mathbf{b}(\tau,\mathrm{i}\eta)\widehat{X} = \widehat{\mathbf{g}} - \mathbf{M}\widehat{\mathbf{U}}(0),
$$

so that we can estimate the front for a generic primitive constant c

$$
\begin{aligned}
(\|\tau\|^2+\|\eta\|^2)\cdot|\widehat{X}|^2 &\leq c(\|\tau\|^2+\|\eta\|^2+s^2\|\eta\|^4)\cdot|\widehat{X}|^2\\
&\leq c\left(\|\widehat{\mathbf{g}}\|^2+\|\widehat{\mathbf{U}}(0)\|^2\right).
\end{aligned}
$$

We note that the mapping

$$
m(\tau,\eta,\zeta):\begin{cases}\mathbb{C}\times\mathcal{E}^s(\tau,\eta) & \to\mathbb{C}^{d+2}\\ (\chi,\mathbf{U}) & \mapsto \chi\,\mathbf{b}(\tau,\eta,\zeta)+\mathbf{M}\mathbf{U}\end{cases}
$$

is one-to-one and onto, such that the standard theory allows us to construct a Kreiss symmetrizer as in the uniform Lopatinski condition with homogenous boundary conditions. Altogether we get the estimate

$$
\begin{aligned}
\gamma\|\tilde{\mathbf{U}}\|^2_{L^2(\mathbb{R}^d\times\mathbb{R}^+)} + \|\tilde{\mathbf{U}}(0)\|^2_{L^2(\mathbb{R}^d)} + \|\tilde{X}\|^2_{H^1_\gamma(\mathbb{R}^d)}&\\
\leq C\left(\|\tilde{\mathbf{g}}\|^2_{L^2(\mathbb{R}^d)} + \frac{1}{\gamma}\|\tilde{\mathbf{f}}\|^2_{L^2(\mathbb{R}^d\times\mathbb{R}^+)}\right)&
\end{aligned}
$$

as stated in Theorem 8.

5 Conclusions

In this part we give further comments on possible extensions of the preceding analysis. The energy estimate given in Theorem 8 is valid only for small surface tension $s<s_0$ and interfacial mobility constant $\mathrm{B}<\mathrm{B}_0$. In particular it gives no statement for small B and arbitrary s which appears to be a stable setting in view of the results in [6]. The difficulty in this context is that η_0 is not given explicitly and therefore it is difficult to estimate η_0 to s_0. The situation for arbitrary value of B is

not so clear with respect to the expected stability.
It has also been suggested that the mobility constant B in the kinetic relation depends on the pressure, as in [10], i.e.

$$\left[g + \frac{j^2}{2\rho^2}\right] = -\mathrm{B}\ (p(\rho_l))^{-1}\ j.$$

In our case where a van der Waals law is included, the linearized system changes to a more complicated one. The associated Lopatinskiĭ determinant takes a structure such that the perturbation analysis is not directly transferable.

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Bugra Kabil
Pfaffenwaldring 57
70569 Stuttgart
Germany
E-Mail: `Bugra.Kabil@mathematik.uni-stuttgart.de`
WWW: `http://www.mathematik.uni-stuttgart.de/fak8/ians/lehrstuhl/LstAngMath/mitarbeiter/kabil/`

Christian Rohde
Pfaffenwaldring 57
70569 Stuttgart
Germany
E-Mail: `Christian.Rohde@mathematik.uni-stuttgart.de`
WWW: `http://www.mathematik.uni-stuttgart.de/fak8/ians/lehrstuhl/LstAngMath/mitarbeiter/rohde/`

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